# Measurement of the intrinsic linewidth of terahertz quantum cascade lasers using a near- infrared frequency comb

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# Measurement of the intrinsic linewidth of terahertz quantum cascade lasers using a near-infrared frequency comb

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**Abstract:** We report the measurement of the frequency noise power spectral density of a quantum cascade laser emitting at 2.5THz. The technique is based on heterodyning the laser emission frequency with a harmonic of the repetition rate of a near-infrared laser comb. This generates a beatnote in the radio frequency range that is demodulated using a tracking oscillator allowing measurement of the frequency noise. We find that the latter is strongly affected by the level of optical feedback, and obtain an intrinsic linewidth of ~230Hz, for an output power of 2mW.



**OCIS codes:** (140.5965) Semiconductor lasers, quantum cascade; (030.1640) Coherence; (140.3070) Infrared and far-infrared lasers.

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18. The ratio $2.6 \times 10^{-3}/0.08 = 0.032$ between the ε coefficients at minimum and maximum isolation corresponds to the field amplitude isolation, i.e. to a power isolation of $1 \times 10^{-3}$, or 30dB. By directly measuring the performance of our isolator using a power detector we found instead an isolation of 16dB. There are several possible explanations for this large difference. The most likely is related to the thickness of the quartz wave plate (3.1+/−0.005mm) being much larger that the QCL wavelength. Therefore, given the QCL large free spectral range of 16GHz, the amount of isolation is strongly dependent on the Fabry-Perot lasing mode number, which can change depending on the feedback conditions. For technical reasons the direct measurement was performed with the QCL operating in pulsed mode, thus lasing on several longitudinal modes, which underestimates the isolation. Instead the QCL was lasing in continuous wave on a single-mode when we measured the frequency pulling.
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## 1. Introduction

THz Quantum cascade lasers (QCLs) are compact and powerful sources with potential applications in imaging, sensing and high-resolution spectroscopy [1]. Recent progress includes the realization of a QCL emitting at 3.9THz at a heat sink temperature of 189K, and the demonstration of active mode-locking with the generation of 10ps-long pulses at 2.5THz [2,3]. The exploitation of these sources can further benefit from investigation and improvement of their coherence properties. In this context frequency stability measurements using mixing techniques [4,5], as well as sub-hertz linewidths by phase-locking to a stable reference [3,6,7] were reported so far, suggesting that THz QCLs are intrinsically narrow linewidth lasers. This is in agreement with their expected small Henry alpha factor stemming from their symmetric gain spectrum, typical of intersubband transitions [8]. An alpha factor of ~0.5 was reported recently by applying a self mixing technique to a 2.6THz QCL [9]. This value is approximately a factor of 10 lower than those of typical interband diode lasers, and should lead to intrinsic linewidth limits in the 100Hz to 10Hz range for power levels from 1 to 10mW. The frequency noise power spectral density (FNSD) of a QCL operating in the THz range was characterized recently by using the slope of the absorption profile of a molecular transition as frequency discriminator [10]. Between 8 and 60MHz, a white noise plateau was observed in the FNSD that was attributed to the laser quantum noise limit, leading to an intrinsic linewidth of ~100Hz.

In this work we exploit an alternative technique based on an electrooptic modulator that allows the generation of an heterodyne beatnote between the emission frequency of a THz QCL and a harmonic of the repetition rate of a near-IR laser comb [6]. The frequency fluctuations of such beatnote are then converted into a voltage signal using a voltage-controlled oscillator (VCO) that tracks the beatnote frequency. Compared to the use of a molecular transition as frequency discriminator, this method is intrinsically broadband, i.e. it can be used to measure the FNSD of THz QCLs at any frequency demonstrated to date. In the range between 20kHz and 300kHz, we observed a white noise plateau that corresponds to the quantum limited QCL FNSD influenced by the effect of optical feedback. By changing the magnitude of the feedback using an optical isolator, the level of the white noise could be

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changed by approximately 1 order of magnitude, leading to linewidths between 30Hz and 500Hz.

## 2. Experiment

The QCL used in our experiment operates at 2.5THz and is based on a standard 2.5mm-long, 240µm-wide ridge-waveguide Fabry-Perot cavity that was fabricated in-house by optical-lithography and wet-etching (details on the waveguide and active region design can be found in Ref [11].). As a near-IR laser comb we exploited a frequency doubled mode-locked fs-fiber laser (Menlo Systems, M-fiber) operating at $\lambda$ = 780nm and emitting a train of ~100fs-long pulses at a repetition rate of ~250MHz (the repetition rate can be tuned by approximately 2 MHz by changing the laser cavity length through a piezoelectric transducer).

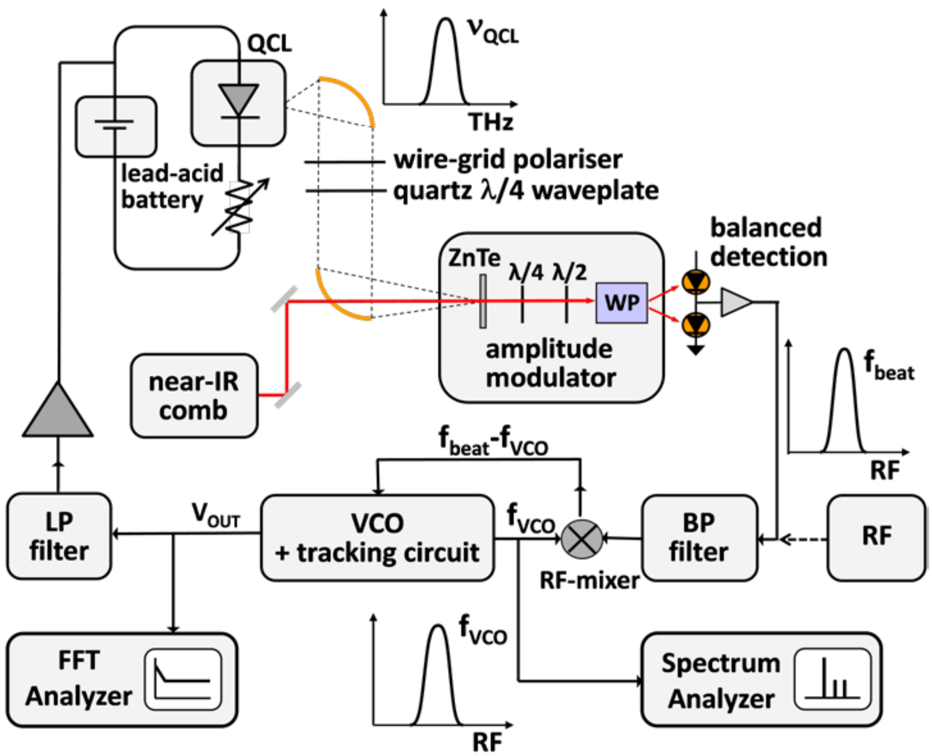


Fig. 1. Experimental setup (see text). At the output of the balanced detection $f_{beat}$ is of the order of a few tens of MHz. Before the band pass filter (labeled BP) $f_{beat}$ is summed to the frequency of an RF synthesiser (not shown) in order to bring it close to 140MHz, the frequency of operation of the VCO. THz optical isolation is obtained as follows. A wire-grid polarizer is oriented parallel to the TM-polarized light from the QCL. Next a quartz quarter-waveplate is oriented with its fast-axis at 45deg with respect to the polarizer, so that at its output the THz light is left circularly polarized. The polarization is changed from left to right circular after reflection from the ZnTe crystal, producing, after the quarter-waveplate, a linearly polarized beam ideally oriented at 90deg with respect to the wire grid polarizer.

The experimental setup for the measurement of the FNSD is shown in Fig. 1. The QCL and near-IR comb beams are collinearly focused on a 2mm-thick, <1,-1,0> oriented ZnTe crystal. The crystal is followed by a pair of $\lambda/4$ and $\lambda/2$ waveplates and a polarizing beam splitter that, together, form an ultrafast near-IR electro-optical amplitude modulator, driven by the THz electric-field amplitude (see Ref [6]. for details on the operating principle). As a result the amplitude of the linearly polarized near-IR comb beam is modulated at 2.5THz generating two sideband combs (assuming that the QCL is single mode) that have a ~50%

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spectral overlap with the carrier comb since the latter has a bandwidth equal to approximately twice the QCL frequency. This generates an heterodyne beatnote oscillating at:

$$f_{beat}(t) = \nu_{QCL}(t) - n \times f_{rep}(t) \tag{1}$$

where $\nu_{QCL}(t)$ is the instantaneous emission frequency of the QCL, $f_{rep}(t)$ is the fs-laser repetition rate and $n = \mathrm{Int}(\nu_{QCL}(t)/f_{rep}) \sim 10^4$ [3,6,7]. Therefore we have that $f_{beat}(t) < f_{rep}/2$ ~125MHz and the beatnote can be detected using a shot-noise limited, balanced detection unit based on a pair of Si-photodiodes (see Fig. 1) [6]. From the equation above, we have that up to the extent where the condition $n \times |df_{rep}(t)/dt| << |d\nu_{QCL}(t)/dt|$ is valid, then $d\nu_{QCL}(t)/dt = df_{beat}(\mathrm{t})/dt$, i.e. the fluctuations of $f_{beat}(\mathrm{t})$ are those of the QCL emission frequency (from now on we will assume that the condition is satisfied, therefore $f_{rep}(t) = f_{rep}$). We note that the present beatnote generation technique is completely insensitive to fluctuations of the QCL amplitude.

The QCL FNSD can be derived from the beatnote signal by using a suitable demodulation technique. In this work we have used a "tracking oscillator" technique that is illustrated schematically in the bottom part of Fig. 1. After passing through a 10MHz-wide bandpass filter, $f_{beat}(\mathrm{t})$ is compared, using an RF mixer, to the frequency of a fast tuning VCO, $f_{VCO}(\mathrm{t})$ of about 140MHz. Next, through a home-made circuit, $f_{VCO}(\mathrm{t})$ is phase-locked to $f_{beat}(t)$ with a bandwidth of 1 to 3MHz. Therefore the VCO correction voltage $V_{out}(t)$, with a slope $\delta V_{out}/\delta f_{beat} = (3.8\mathrm{MHz/V})^{-1}$, represents the sum of the VCO and $f_{beat}(t)$ frequency noise for Fourier frequencies below the locking bandwidth (~1-3 MHz). However the VCO phase/frequency noise is negligible compared to that of $f_{beat}(\mathrm{t})$. Such a frequency discriminator tracks frequency deviations up to ~5-10MHz. As shown in Fig. 1, a fraction of $V_{out}(t)$ is used to control the QCL current through a "slow" control loop with a bandwidth below 1kHz. This is necessary to eliminate the QCL low Fourier frequencies noise and drift (few MHz/s) produced by thermal and mechanical fluctuations, and thus maintain $f_{beat}(\mathrm{t})$ within the 5-10MHz tracking range [5]. The power spectrum of $V_{out}(t)$ is then measured using a fast-Fourier transform analyzer (FFT in Fig. 1), and the FNSD of the QCL is finally derived through the measured VCO slope.

For the experiment the QCL was Indium-bonded on a copper holder, which was mounted on the cold head of a continuous-flow liquid Helium cryostat. The QCL operating temperature was stabilized at 20K, and the laser was driven in continuous wave at currents between 1.1 and 1.3A using a lead-acid battery for minimum noise.

An example of measured FNSD is shown by the red curve of Fig. 2, in the range 100Hz-1MHz (note that below ~10kHz the red curve overlaps almost completely with the black one). Four spectral regions can be clearly distinguished. At frequencies below ~300Hz the frequency noise curve is dominated by the slow frequency lock of the QCL. Above ~300Hz the frequency noise rolls-off, with a slope roughly proportional to $1/f^{\,2}$ in the range 1kHz-10kHz. This excess technical noise can be attributed to environmental parameters such as mechanical vibrations (see below) or temperature/current fluctuations [12]. In the range 10kHz-100kHz we observe a flat plateau at ~75Hz$^2$/Hz that we identify as the quantum noise-limit of the QCL FNSD (see the next Section for discussion). Above 100kHz the frequency noise grows up to approximately 1MHz, beyond which we observe a decrease due to the finite bandwidth of the tracking circuit. The increase of the frequency-noise above100kHz is the consequence of our shot-noise limited detection noise floor, resulting into a white phase noise power density in the carrier (i.e. $f_{beat}(\mathrm{t})$) frequency domain. This gives rise to a double-sided FNSD given by (2/SNR) $\times f^{\,2}$, where SNR is the Signal-to-Noise Ratio (or phase noise power density to carrier ratio) in 1Hz bandwidth at the output of the balanced detection [13]. To verify that this is indeed the case, the blue curve in Fig. 2 shows the FNSD obtained by blocking the the THz QCL beam, and by replacing $f_{beat}$ with the signal generated by a

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synthesizer (labeled "RF" in Fig. 1), with an output power yielding a SNR of + 89dB/Hz, i.e. exactly equal to that of $f_{beat}$ when the red trace was recorded (this condition was carefully verified using the spectrum analyzer shown in Fig. 1). As shown by the computed dashed line in Fig. 2, in the range ~40kHz-400kHz the finite SNR gives the expected frequency noise of 2 x $10^{-8.9} \times f^{2}$. The effect of the finite SNR on the recorded QCL FNSD can be partially removed by normalizing to the synthesizer trace. This is shown by the black trace in Fig. 2, obtained by calculating the ratio between the red curve and the blue one. This normalization process extends up to ~300kHz the white noise plateau of the QCL (at higher frequencies the normalization becomes meaningless since the difference between the two traces is comparable to the noise of each single trace).

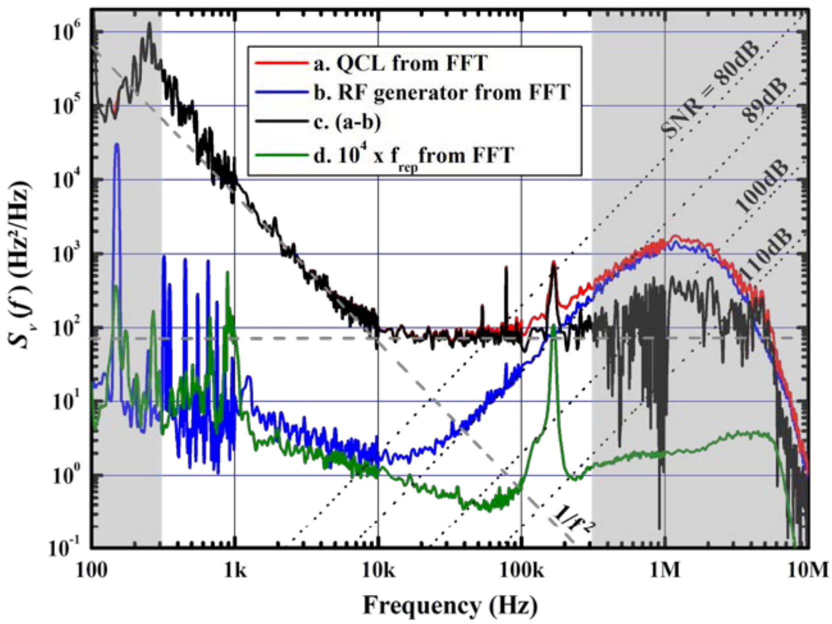


Fig. 2. FNSD ($S_\nu(f)$) traces measured with the FFT analyser of the QCL (red), the RF generator (blue), and of $n \times f_{rep} \sim 10^{4} \times f_{rep}$ (green). The black line is obtained by dividing the red line by the blue line, and represents the FNSD of the QCL normalised to the frequency noise produced by the detection system noise floor (see text). The black dotted lines are given by the relation $S_\nu(f) = (2/\mathrm{SNR}) \times f^{2}$, for SNRs of 80, 89, 100, and 110dB in 1Hz bandwidth. The horizontal grey line is a guide to the eye indicating the quantum noise limited QCL FNSD. Shaded grey areas correspond to regions where the black line does not correspond to the QCL FNSD. We attribute the peak close to 167kHz in the green curve to a spurious frequency noise of the near-IR comb produced by stray oscillations of the power-supply. The level of the peak is mode-locking state dependent. This explains the discrepancy between the peaks in the black and green curves.

To verify that, as discussed above, the residual noise of $n \times f_{rep}$ is negligible compared to the QCL frequency noise, we have measured the noise of the lowest frequency beating of the near-IR comb with a very narrow (< 3kHz) continuous-wave fibre laser at 1542nm (195THz). This means measuring the frequency noise of $m \times f_{rep}$ with $m \approx \mathrm{Int}(195\mathrm{THz}/f_{rep}) \approx 8 \cdot 10^{5}$. The green trace in Fig. 2 shows the result of such measurement multiplied by the factor $(n/m)^2 \approx (2.5/195)^2$, i.e. the FNSD of $n \times f_{rep}$. As can be seen, except for a peak at 167kHz (see the caption of Fig. 2), the noise is at least two decades below the frequency noise of the QCL. We note that the green trace overestimates the actual noise of $f_{rep}$ since it includes also the noise of the comb offset frequency. Finally, we have also verified that the current noise due to the battery used to drive the QCL is below the noise floor of our measuring system, leading to a

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frequency noise contribution at least two orders of magnitude below the experimental QCL FNSD.

### 3. Effect of optical feedback and intrinsic linewidth

During the measurements we found that changing the distance between the ZnTe crystal and the QCL produced a change of $f_{beat}$. This is due to an unwanted optical feedback caused by the reflection from the ZnTe, effectively forming a 25cm-long external cavity with the QCL facet. As shown in Fig. 1 (see also the Figure caption), to establish the effect of this feedback on the QCL frequency noise we used a wire grid polarizer followed by a quartz quarter-wave plate to realize a simple optical isolator. In Fig. 3 we report four traces recorded at output powers (drive currents) of 0.8mW(1.12A) and 2.0mW(1.3A) respectively, with the QCL always emitting in a single mode (the solid black trace is the same as the black trace of Fig. 2). The traces were obtained using the normalization procedure previously described. For each current level the FNSD QCL was first measured with minimum (dotted curves) and maximum (solid curves) optical isolation. As shown in Fig. 3, above ~10kHz the frequency noise is reduced by approximately 10dB when the isolation is removed, which we attribute to the effect of self-injection locking [14–17].

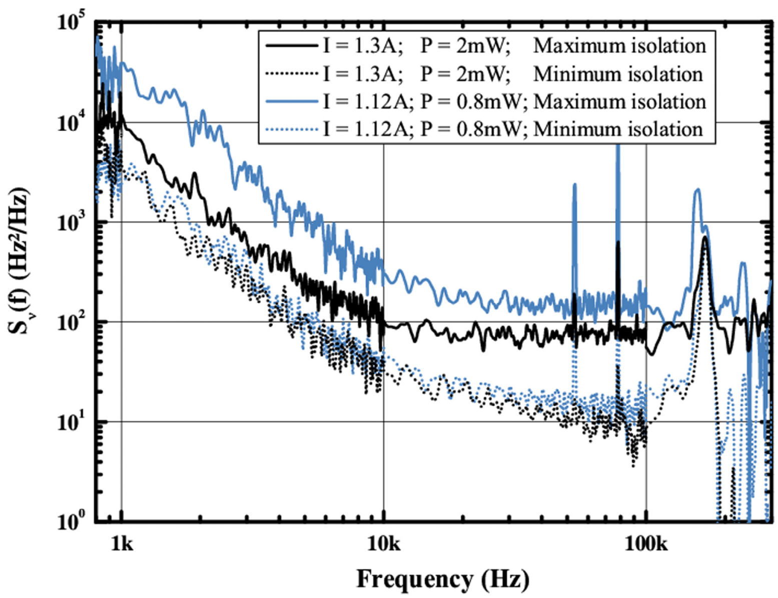


Fig. 3. FNSD curves of the QCL for output powers/currents of 2mW/1.3A (black) and 0.8mW/1.12A (blue), and maximum (solid) and minimum isolation (dotted). The output power corresponds to the power emitted by one QCL facet and was measured with a calibrated THz power meter. All the curves were normalised to the FNSD produced by the detection system noise floor (see text and the caption of Fig. 2). The solid black trace is the same as the black trace of Fig. 2.

When light is fed back in the cavity of the QCL two effects are expected: a frequency pulling and a change of the linewidth. The frequency pulling is given by the following equation [16,17]:

$$v_{QCL} - v_0 = -\frac{cK}{4\pi L_{ext}}[\sin(4\pi v_{QCL} L_{ext} / c) + \alpha_H \cos(4\pi v_{QCL} L_{ext} / c)] \tag{2}$$

with the feedback coupling coefficient $K$ defined as

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$$K = \varepsilon(1 - R_{QCL})\sqrt{\frac{R_{ZnTe}}{R_{QCL}}}\frac{L_{ext}}{L_{QCL}n_{eff}} \tag{3}$$

Here $R_{QCL}$ (0.3) and $R_{ZnTe}$ (0.3) are the reflectivities of the QCL facet and ZnTe crystal; $\nu_0$ is the QCL free-running frequency (i.e. without feedback); $\alpha_H$ is the Henry alpha-factor; $n_{eff}$ = 3.5 is the effective index of the QCL lasing mode; $L_{ext}$ = 25cm and $L_{QCL}$ = 2.5mm are the length of the external cavity and of the QCL ridge respectively, and the coupling parameter $\varepsilon$ takes into account all additional attenuation factors, including the fraction of the reflected field that couples back coherently into the lasing mode [16]. Assuming that $\alpha_H$ is negligible, when the feedback coefficient $K$ is greater than 1 Eq. (2) has more than one solution, leading to mode jumps. Instead, for values of the feedback such that $K< 1$, Eq. (2) has only one solution, and the QCL frequency oscillates periodically with $L_{ext}$ with a period given by half the free-space wavelength. In this case an estimate of $K$ can be obtained by monitoring the pulling of $f_{beat}$ as a function of the distance between the ZnTe crystal and the QCL facet [16].

With minimum isolation we observed mode jumps indicating that $K > 1$. At maximum isolation we found instead a continuous periodic modulation of $f_{beat}$ with a period of 60 μm, i.e. equal to half the emission wavelength of the QCL, and a peak-to-peak amplitude of 10MHz, (I = 1.3A, see the black solid trace in Fig. 3). From Eq. (3) with $\alpha_H = 0$ this gives a feedback coupling coefficient $K$ ~0.05 ($\varepsilon = 2.6\text{x}10^{-3}$).

The ratio between the linewidth with feedback, $\Delta\nu$, and the linewidth with no feedback (i.e. the QCL intrinsic linewidth), $\Delta\nu_0$, is given by [17]:

$$\Delta\nu = \frac{\Delta\nu_0}{\left\{1+\sqrt{1+\alpha_H^2}\,K\cos[(4\pi\nu_{QCL}L_{ext}/c)+tg^{-1}\alpha_H)]\right\}^2} \tag{4}$$

This formula essentially translates the fact that, depending on the phase of the feedback light, the observed linewidth can be above or below the intrinsic laser linewidth, with a modulation depth determined by $K$. In the case of maximum isolation, from the derived $K$~0.05 and assuming that $\alpha_H = 0$, we have that $0.9 < (\Delta\nu/\Delta\nu_0) < 1.1$ (note that if $\alpha_H > 0$ the linewidth modulation would be even lower). Therefore, given the experimental error, $\Delta\nu$ is equal to the intrinsic QCL linewidth. From the 75Hz$^2$/Hz white noise plateau of the solid black trace in Fig. 3, we obtain $\Delta\nu_0$~235Hz~π×75Hz for an output power per facet of 2mW (the power was measured with a calibrated THz power meter) [12].

With minimum isolation $K > 1$, and $\Delta\nu$ can be substantially different from $\Delta\nu_0$. In this condition a lower limit for $K$ can be derived (with $\alpha_H$ ~0) from Eq. (4), and by considering the ratio between the white noise plateau of the solid black trace ($K$ ~0.05) and that of the dotted black trace of Fig. 3. From the measured ratio of ~7 we obtain $K > 1.65$ which is consistent with the observation of mode jumps. Using Eq. (3) this leads to $\varepsilon > 0.08$. This factor includes the *field amplitude* attenuation through the high density polyethylene window of our cryostat and of the quartz waveplate, both with a ~0.7 transmission coefficient. By normalizing $\varepsilon$ to these values we obtain a fractional *field* coupling of the reflected mode into the QCL lasing mode of 16%, corresponding to a *power* coupling of 2.5% = $(0.16)^2$ [18].

The obtained intrinsic linewidth of $\Delta\nu_0$ ~235Hz for an output power per facet of 2mW can be compared with the expected linewidth of a semiconductor laser, given by the following well-known expression:

$$\Delta\nu_0 = \left(\frac{c}{n'_{eff}}\right)^2 \frac{h\nu_{QCL}\alpha_t\alpha_m(1+\alpha_H^2)}{8\pi P} \tag{5}$$

where $h$ is the Planck constant, $n'_{eff}$ is the effective group refractive index, $\alpha_t = 10\text{cm}^{-1}$ and $\alpha_m = 5\text{cm}^{-1}$ are the QCL total and radiative losses, and $P$ is the output power from one QCL facet

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[19]. $n'_{eff}$ can be derived from the free spectral range of a multimode QCL with the same waveguide as the one used in this work. The latter was determined with very high accuracy in Ref [20] by measuring the heterodyne beatnote between longitudinal Fabry-Perot modes, yielding $n'_{eff} = 3.75$. From Eq. (5) with $P = 2$mW, we therefore obtain $\Delta\nu_0 = 105$Hz in the case where $\alpha_H = 0$, which is approximately a factor of 2 below the linewidth derived from the measured FNSP. Although this finding is compatible with the experimental errors it could suggest that $\alpha_H$ is actually non-negligible but rather close to 1 [9]. Recently it is has been argued that an additional broadening of the linewidth of THz QCLs occurs from photons generated by blackbody radiation in the laser active region [21]. However the estimated correction (<50%, see Ref [10].) is within the error of our measurement, and moreover its value is known with a high uncertainty. Therefore more systematic measurements should be performed to verify this hypothesis.

## 4. Conclusions

In conclusion we have presented an original technique that allows the measurement of the FNSP of THz QCLs. The technique is based on the generation of a beatnote signal between the QCL frequency and the repetition rate of a near-IR frequency comb [6,7]. As such it is potentially applicable to any QCL, provided that its emission frequency falls within the comb spectral bandwidth. With a QCL operating on a single mode at 2.5THz and emitting a power of 2mW we obtain a quantum noise limited linewidth of 230Hz, showing that THz QCLs are ultra-narrow linewidth lasers thanks to their very small Henry alpha-factor [9]. Similar values of the linewidth have been obtained with another device fabricated from the same wafer. These findings are in agreement with the results of Ref [10], obtained with a completely different technique.

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